

# A phenomenological equation of state for isospin asymmetric nuclear matter

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A phenomenological momentum-independent (MID) model is constructed to describe the equation of state (EOS) for isospin asymmetric nuclear matter, especially the density dependence of the nuclear symmetry energy  $E_{\text{sym}}(\rho)$ . This model can reasonably describe the general properties of the EOS for symmetric nuclear matter and the symmetry energy predicted by both the sophisticated isospin and momentum dependent MDI model and the Skyrme-Hartree-Fock approach. We find that there exists a nicely linear correlation between  $K_{\text{sym}}$  and  $L$  as well as between  $J_0/K_0$  and  $K_0$ , where  $L$  and  $K_{\text{sym}}$  represent, respectively, the slope and curvature parameters of the symmetry energy at the normal nuclear density  $\rho_0$  while  $K_0$  and  $J_0$  are, respectively, the incompressibility and the third-order derivative parameter of symmetric nuclear matter at  $\rho_0$ . These correlations together with the empirical constraints on  $K_0$ ,  $L$  and  $E_{\text{sym}}(\rho_0)$  lead to an estimation of  $-477 \text{ MeV} \leq K_{\text{sat},2} \leq -241 \text{ MeV}$  for the second-order isospin asymmetry expansion coefficient for the incompressibility of asymmetric nuclear matter at the saturation point.

PACS numbers: Equation of state of nuclear matter, isospin, the symmetry energy

## I. INTRODUCTION

The study of the isospin degree of freedom in nuclear physics has recently attracted much attention due to the establishment of many radioactive beam facilities around the world. Besides the many existing radioactive beam facilities and their upgrades, such as the Cooling Storage Ring (CSR) facility at HIRFL in China [1], many more are being constructed or under planning, including the Radioactive Ion Beam (RIB) Factory at RIKEN in Japan [2], the FAIR/GSI in Germany [3], SPIRAL2/GANIL in France [4], and the Facility for Rare Isotope Beams (FRIB) in the USA [5]. These new facilities offer the possibility to study the properties of nuclear matter or nuclei under the extreme condition of large isospin asymmetry. The ultimate goal of such study is to extract information on the isospin dependence of in-medium nuclear effective interactions as well as the equation of state (EOS) of isospin asymmetric nuclear matter, particularly its isospin-dependent term or the density dependence of the nuclear symmetry energy. This knowledge, especially the latter, is important for understanding not only the structure of radioactive nuclei, the reaction dynamics induced by rare isotopes, and the liquid-gas phase transition in asymmetric nuclear matter, but also many critical issues in astrophysics [6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

The EOS of nuclear matter is one of fundamental questions in nuclear physics. For symmetric nuclear matter, the EOS is relatively well-determined after about more than 30 years of studies in the nuclear physics community. The incompressibility of symmetric nuclear matter at its saturation density  $\rho_0$  has been determined to be  $240 \pm 20 \text{ MeV}$  from the nuclear giant monopole resonances (GMR) [16, 17, 18, 19, 20] and the EOS at densities of  $2\rho_0 < \rho < 5\rho_0$  has also been constrained by measurements of collective flows in nucleus-nucleus collisions [8] and of sub-threshold kaon production [21, 22] in relativistic nucleus-

nucleus collisions. On the other hand, for asymmetric nuclear matter, the EOS, especially the density dependence of the nuclear symmetry energy, is largely unknown. Although the nuclear symmetry energy at  $\rho_0$  is known to be around 30 MeV from the empirical liquid-drop mass formula [23, 24], its values at other densities are poorly known [6, 7]. Various microscopic and phenomenological models, such as the relativistic Dirac-Brueckner-Hartree-Fock (DBHF) [25, 26, 27, 28, 29, 30, 31] and the non-relativistic Brueckner-Hartree-Fock (BHF) [32, 33, 34] approach, the relativistic mean-field (RMF) model based on nucleon-meson interactions [12, 35, 36, 37], and the non-relativistic mean-field model based on Skyrme-like interactions [38, 39, 40, 41, 42, 43, 44, 45], have been used to study the isospin-dependent properties of asymmetric nuclear matter, such as the nuclear symmetry energy, the nuclear symmetry potential, the isospin-splitting of nucleon effective mass, etc., but the predicted results vary widely. In fact, even the sign of the symmetry energy above  $3\rho_0$  is uncertain [46, 47]. The theoretical uncertainties are mainly due to the lack of knowledge about the isospin dependence of in-medium nuclear effective interactions and the limitations in the techniques for solving the nuclear many-body problem.

In the present work, we construct a phenomenological momentum-independent (MID) model which can reasonably describe the general properties of symmetric nuclear matter and the symmetry energy predicted by both the sophisticated isospin and momentum dependent MDI model and the Skyrme-Hartree-Fock approach with different Skyrme forces. In particular, the density functional of the symmetry energy constructed in the MID model is shown to be very flexible and can mimic very different density behaviors by varying only one parameter. We find that there exists a nicely linear correlation between  $K_{\text{sym}}$  and  $L$  as well as between  $J_0/K_0$  and  $K_0$ , where  $L$  and  $K_{\text{sym}}$  represent, respectively, the slope

and curvature parameters of the symmetry energy at the normal nuclear density  $\rho_0$  while  $K_0$  and  $J_0$  are, respectively, the incompressibility and the third-order derivative parameter of symmetric nuclear matter at  $\rho_0$ . These correlations together with the empirical constraints on  $K_0$ ,  $L$  and  $E_{\text{sym}}(\rho_0)$  lead to an estimate of  $-477$  MeV  $\leq K_{\text{sat},2} \leq -241$  MeV for the second-order isospin asymmetry expansion coefficient for the incompressibility of asymmetric nuclear matter at the saturation point, which is presently largely uncertain and being heavily discussed [18, 19, 20, 48, 49, 50].

The paper is organized as follows. In Section II, we discuss general properties of asymmetric nuclear matter, and then introduce the momentum independent MID model. The results and discussions are presented in Section III. A summary is then given in Section IV.

## II. THEORETICAL MODELS AND METHODS

### A. Equation of state of asymmetric nuclear matter

The EOS of isospin asymmetric nuclear matter, given by its binding energy per nucleon, can be expanded to 2nd-order in isospin asymmetry  $\delta$  as

$$E(\rho, \delta) = E_0(\rho) + E_{\text{sym}}(\rho)\delta^2 + O(\delta^4), \quad (1)$$

where  $\rho = \rho_n + \rho_p$  is the baryon density with  $\rho_n$  and  $\rho_p$  denoting the neutron and proton densities, respectively;  $\delta = (\rho_n - \rho_p)/(\rho_p + \rho_n)$  is the isospin asymmetry;  $E_0(\rho) = E(\rho, \delta = 0)$  is the binding energy per nucleon in symmetric nuclear matter, and the nuclear symmetry energy is expressed as

$$E_{\text{sym}}(\rho) = \frac{1}{2!} \frac{\partial^2 E(\rho, \delta)}{\partial \delta^2} \Big|_{\delta=0}. \quad (2)$$

The absence of odd-order terms in  $\delta$  in Eq. (1) is due to the exchange symmetry between protons and neutrons in nuclear matter when one neglects the Coulomb interaction and assumes the charge symmetry of nuclear forces. The higher-order coefficients in  $\delta$  are usually very small and negligible, e.g., the magnitude of the  $\delta^4$  term at normal nuclear density  $\rho_0$  is estimated to be less than 1 MeV in microscopic many-body approaches [51, 52, 53]. Neglecting the contribution from higher-order terms in Eq. (1) leads to the well-known empirical parabolic law for the EOS of asymmetric nuclear matter, which has been verified by all many-body theories to date, at least for densities up to moderate values [15]. As a good approximation, the density-dependent symmetry energy  $E_{\text{sym}}(\rho)$  can thus be extracted from the parabolic approximation of  $E_{\text{sym}}(\rho) \approx E(\rho, \delta = 1) - E(\rho, \delta = 0)$ .

Around the normal nuclear density  $\rho_0$ , the binding energy per nucleon in symmetric nuclear matter  $E_0(\rho)$  can be expanded, e.g., up to 3rd-order in density as

$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2!} \chi^2 + \frac{J_0}{3!} \chi^3 + O(\chi^4), \quad (3)$$

where  $\chi$  is a dimensionless variable characterizing the deviations of the density from the saturation density  $\rho_0$  of the symmetric nuclear matter and it is conventionally defined as  $\chi = (\rho - \rho_0)/3\rho_0$ .  $E_0(\rho_0)$  is the binding energy per nucleon in symmetric nuclear matter at the saturation density  $\rho_0$  and the other coefficients can be calculated as

$$K_0 = 9\rho_0^2 \frac{d^2 E_0(\rho)}{d\rho^2} \Big|_{\rho=\rho_0}, J_0 = 27\rho_0^3 \frac{d^3 E_0(\rho)}{d\rho^3} \Big|_{\rho=\rho_0}. \quad (4)$$

Obviously, there is no linear  $\chi$  term in Eq. (3) according to the definition of the saturation density  $\rho_0$ .  $K_0$  is the incompressibility coefficient of symmetric nuclear matter and it characterizes the curvature of  $E_0(\rho)$  at  $\rho_0$ . The coefficient  $J_0$  corresponds to the third-order derivative parameter of symmetric nuclear matter at  $\rho_0$ . In the literature, people usually neglect the higher-order terms in Eq. (3) and obtain the following parabolic approximation to the EOS of symmetric nuclear matter:

$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2} \chi^2 + O(\chi^3). \quad (5)$$

Similarly, around  $\rho_0$ , the nuclear symmetry energy  $E_{\text{sym}}(\rho)$  can be expanded, e.g., up to 2nd-order in density as

$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + L\chi + \frac{K_{\text{sym}}}{2!} \chi^2 + O(\chi^3),$$

where  $L$  and  $K_{\text{sym}}$  are the slope parameter and curvature parameter of the nuclear symmetry energy at  $\rho_0$ , i.e.,

$$L = 3\rho_0 \frac{dE_{\text{sym}}(\rho)}{d\rho} \Big|_{\rho=\rho_0}, K_{\text{sym}} = 9\rho_0^2 \frac{d^2 E_{\text{sym}}(\rho)}{d\rho^2} \Big|_{\rho=\rho_0}. \quad (6)$$

The coefficients  $L$  and  $K_{\text{sym}}$  characterize the density dependence of the nuclear symmetry energy around normal nuclear density  $\rho_0$ , and thus carry important information on the properties of nuclear symmetry energy at both high and low densities.

The incompressibility is an essential quantity for nuclear matter and conventionally it is defined at the saturation density  $\rho_{\text{sat}}$  for asymmetric nuclear matter where we have  $P(\rho, \delta) = 0$  (the incompressibility coefficient at the saturation density is called isobaric incompressibility coefficient in [54]) and thus it can be expressed as

$$K_{\text{sat}}(\delta) = 9\rho_{\text{sat}}^2 \frac{\partial^2 E(\rho, \delta)}{\partial \rho^2} \Big|_{\rho=\rho_{\text{sat}}}. \quad (7)$$

For asymmetric nuclear matter, the isobaric incompressibility coefficient  $K_{\text{sat}}(\delta)$  can be expressed up to 2nd-order in  $\delta$  as [55]

$$K_{\text{sat}}(\delta) = K_0 + K_{\text{sat},2}\delta^2 + O(\delta^4), \quad (8)$$

with

$$K_{\text{sat},2} = K_{\text{sym}} - 6L - \frac{J_0}{K_0} L. \quad (9)$$

The coefficient  $K_{\text{sat},2}$  essentially reflects the isospin dependence of the isobaric incompressibility of asymmetric nuclear matter.

If we use the parabolic approximation to EOS of symmetric nuclear matter, i.e., Eq. (5), then the  $K_{\text{sat},2}$  is reduced to

$$K_{\text{asy}} = K_{\text{sym}} - 6L \quad (10)$$

and this expression has been extensively used to characterize the isospin dependence of the incompressibility of asymmetric nuclear matter in the literature [56, 57, 58, 59]. Obviously, we have

$$K_{\text{sat},2} = K_{\text{asy}} - \frac{J_0}{K_0}L, \quad (11)$$

and thus the coefficient  $K_{\text{asy}}$  could be a good approximation to  $K_{\text{sat},2}$  if  $J_0$  is negligible or the slope parameter of the symmetry energy  $L$  is very small.

It is believed that information on  $K_{\text{sat},2}$  can in principle be extracted experimentally by measuring the GMR in neutron-rich nuclei [55]. Usually, one can define a finite nucleus incompressibility  $K_A(N, Z)$  for a nucleus with  $N$  neutrons and  $Z$  protons ( $A = N + Z$ ) by the energy of GMR  $E_{\text{GMR}}$ , i.e.,

$$E_{\text{GMR}} = \sqrt{\frac{\hbar^2 K_A(N, Z)}{m \langle r^2 \rangle}}, \quad (12)$$

where  $m$  is the nucleon mass and  $\langle r^2 \rangle$  is the mean square mass radius of the nucleus at ground state. Similar to the semi-empirical mass formula, the finite nucleus incompressibility  $K_A(N, Z)$  can be expanded as

$$K_A(N, Z) = K_0 + K_{\text{surf}}A^{-1/3} + K_{\tau} \left( \frac{N - Z}{A} \right)^2 + K_{\text{Coul}} \frac{Z^2}{A^{4/3}}, \quad (13)$$

where  $K_0$ ,  $K_{\text{surf}}$ ,  $K_{\tau}$ , and  $K_{\text{Coul}}$  represent the volume, surface, symmetry, and Coulomb terms, respectively. The  $K_{\tau}$  parameter is usually thought to be equivalent to the  $K_{\text{sat},2}$  parameter. It should be noted here that the  $K_{\text{sat},2}$  parameter is theoretically a well-defined physical quantity while the value of the  $K_{\tau}$  parameter may depend on the detailed truncations in the expansion similarly to the semi-empirical mass formula. Earlier attempts based on the above method have given widely different values for the  $K_{\tau}$  parameter. For example, a value of  $K_{\tau} = -320 \pm 180$  MeV with a large uncertainty was obtained in Ref. [60] from a systematic study of the GMR in the isotopic chains of Sn and Sm. In this analysis, the value of  $K_0$  was found to be  $300 \pm 25$  MeV, which is somewhat larger than the commonly accepted value of  $240 \pm 20$  MeV. In a later study, an even less stringent constraint of  $-566 \pm 1350 < K_{\tau} < 139 \pm 1617$  MeV was extracted from the GMR of finite nuclei, depending on the mass region of nuclei and the number of parameters used in parameterizing the incompressibility

of finite nuclei [61]. Most recently, a much stringent constraint of  $K_{\tau} = -550 \pm 100$  MeV has been obtained in Ref. [18, 19] from measurements of the isotopic dependence of the GMR in even- $A$  Sn isotopes.

## B. A phenomenological momentum-independent MID model

In the present work, we will mainly use three models, i.e., the isospin and momentum dependent MDI interaction [38], the Hartree-Fock approach based on Skyrme interactions, and a phenomenological momentum-independent interaction (MID). The MDI interaction [38] is based on the finite-range Gogny effective interaction and has been used extensively in the literature [15]. The SHF approach is a well-known mean-field theory and has been extensively used in the literature for its simplicity. A very useful feature of these models is that analytical expressions for many interesting physical quantities in asymmetric nuclear matter at zero temperature can be obtained. Here we only introduce the MID model and for the MDI and SHF models, one can refer to, e.g., Refs. [38, 62].

In the momentum-independent MID model, following the results from SHF approach with the zero-range and momentum-independent Skyrme interaction, the potential energy density  $V_{\text{MID}}(\rho, \delta)$  of a cold symmetric nuclear matter at total density  $\rho$  and isospin asymmetry  $\delta$  is parametrized as

$$V_{\text{MID}}(\rho, \delta) = \frac{\alpha}{2} \frac{\rho^2}{\rho_0} + \frac{\beta}{\sigma + 1} \frac{\rho^{\sigma+1}}{\rho_0^{\sigma}} + \rho E_{\text{sym}}^{\text{pot}}(\rho) \delta^2. \quad (14)$$

In the MID model, the 4th-order and higher-order nuclear symmetry energy are not included and we assume they can be negligible. The parameters  $\alpha$ ,  $\beta$  and  $\sigma$  are determined by the binding energy per nucleon  $E_0(\rho_0) = -16$  MeV and the incompressibility  $K_0$  at the saturation density  $\rho_0 = 0.16 \text{ fm}^{-3}$

$$\alpha = -29.47 - 46.74 \frac{K_0 + 44.21}{K_0 - 166.11} \text{ (MeV)}, \quad (15)$$

$$\beta = 23.37 \frac{K_0 + 254.53}{K_0 - 166.11} \text{ (MeV)}, \quad (16)$$

$$\sigma = \frac{K_0 + 44.21}{210.32}, \quad (17)$$

where the unit of  $K_0$  is MeV.

For the potential part of the symmetry energy  $E_{\text{sym}}^{\text{pot}}(\rho)$  in the MID model, it is parametrized as

$$E_{\text{sym}}^{\text{pot}}(\rho) = E_{\text{sym}}^{\text{pot}}(\rho_0)(1 - y) \frac{\rho}{\rho_0} + y E_{\text{sym}}^{\text{pot}}(\rho_0) \left( \frac{\rho}{\rho_0} \right)^{\gamma_{\text{sym}}} \quad (18)$$

with  $E_{\text{sym}}^{\text{pot}}(\rho_0) = E_{\text{sym}}(\rho_0) - E_{\text{sym}}^{\text{kin}}(\rho_0) = 17.7$  MeV following  $E_{\text{sym}}(\rho_0) = 30$  MeV and  $E_{\text{sym}}^{\text{kin}}(\rho_0) = \frac{\hbar^2}{6m} \left( \frac{3\pi^2}{2} \rho_0 \right)^{2/3} = 12.3$  MeV. The default value of the

$\gamma_{\text{sym}}$  parameter is taken to be  $4/3$  in the MID model following the  $E_{\text{sym}}(\rho)$  in the MDI interaction (we will see how the  $\gamma_{\text{sym}}$  parameter affects the symmetry energy in the following). Similarly to the  $x$  parameter introduced in the MDI interaction [58], the dimensionless  $y$  parameter is introduced to mimic various  $E_{\text{sym}}(\rho)$  predicted by different microscopic and/or phenomenological many-body theories for a fixed  $\gamma_{\text{sym}}$  parameter. As we will show later, for  $\gamma_{\text{sym}} = 4/3$ , adjusting the  $y$  value can nicely reproduce the  $E_{\text{sym}}(\rho)$  in the MDI interaction with  $x = -1, 0$ , and  $1$ .

In the MID model, the EOS of symmetric nuclear matter can thus be written as

$$E_0(\rho) = \frac{3\hbar^2}{10m} \left( \frac{3\pi^2}{2}\rho \right)^{2/3} + \frac{\alpha}{2} \frac{\rho}{\rho_0} + \frac{\beta}{\sigma+1} \left( \frac{\rho}{\rho_0} \right)^\sigma, \quad (19)$$

and the symmetry energy can be expressed as

$$\begin{aligned} E_{\text{sym}}(\rho) = & \frac{\hbar^2}{6m} \left( \frac{3\pi^2}{2}\rho \right)^{2/3} \\ & + [E_{\text{sym}}(\rho_0) - E_{\text{sym}}^{\text{kin}}(\rho_0)](1-y) \frac{\rho}{\rho_0} \\ & + y[E_{\text{sym}}(\rho_0) - E_{\text{sym}}^{\text{kin}}(\rho_0)] \left( \frac{\rho}{\rho_0} \right)^{\gamma_{\text{sym}}} \end{aligned} \quad (20)$$

which leads to

$$L = 2E_{\text{sym}}^{\text{kin}}(\rho_0) + 3[E_{\text{sym}}(\rho_0) - E_{\text{sym}}^{\text{kin}}(\rho_0)] + 3y(\gamma_{\text{sym}} - 1)[E_{\text{sym}}(\rho_0) - E_{\text{sym}}^{\text{kin}}(\rho_0)] \quad (21)$$

$$K_{\text{sym}} = 9y\gamma_{\text{sym}}(\gamma_{\text{sym}} - 1)[E_{\text{sym}}(\rho_0) - E_{\text{sym}}^{\text{kin}}(\rho_0)] - 2E_{\text{sym}}^{\text{kin}}(\rho_0). \quad (22)$$

### III. RESULTS AND DISCUSSIONS

#### A. The nuclear symmetry energy and correlation between $L$ and $K_{\text{sym}}$

As mentioned above, in the MID model, the density dependence of the symmetry energy can be adjusted by varying the  $y$  parameter as shown in Eq. (20). As an example, we show in Figure 1 the density dependence of the symmetry energy from the MID interaction with  $y = -3.4, -0.73$ , and  $1.8$ . The corresponding results from the MDI interaction with  $x = 1, 0$ , and  $-1$  as well as the widely used APR (Akmal-Pandharipande-Ravenhall) prediction [63] are also included for comparison. Indeed, one can see that the MID interaction can give a nice description on the density dependence of the symmetry energy predicted by the sophisticated MDI interaction from the very soft ( $x = 1$ ) to the very stiff one ( $x = -1$ ). Furthermore, it is seen that the APR prediction for the symmetry energy at subsaturation densities lies right between that with  $x = 0$  and  $-1$ , and especially the symmetry energy with  $x = 0$  (and  $y = -0.73$ ) resembles very

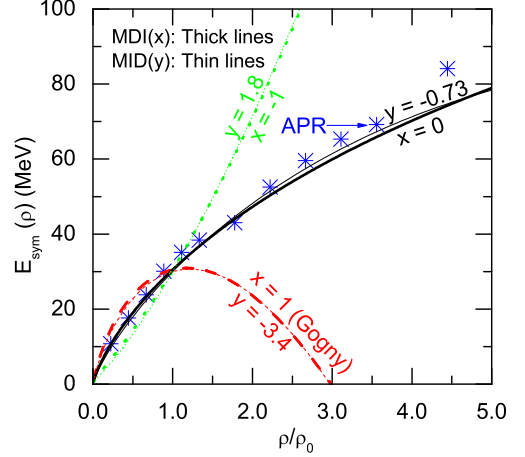


FIG. 1: (Color online) Density dependence of the symmetry energy from the phenomenological MID interaction prediction with  $y = -3.4, -0.73$ , and  $1.8$ . The results from the MDI interaction with  $x = 1, 0$ , and  $-1$  and the widely used APR (Akmal-Pandharipande-Ravenhall) prediction [63] are also included for comparison.

well the APR prediction up to about  $3.5\rho_0$ . These features imply that the density functional of the symmetry energy shown in Eq. (20) is very flexible and can give a quite general description for density dependence of the symmetry energy.

The parameters  $L$  and  $K_{\text{sym}}$  are determined by the density dependence of the symmetry energy around  $\rho_0$ . In recent years, significant progress has been made both experimentally and theoretically in extracting the information on the symmetry energy at sub-saturation density from heavy-ion reactions. Using the isospin and momentum-dependent IBUU04 transport model with in-medium NN cross sections, the isospin diffusion data were found to be consistent with the symmetry energy from the MDI interaction with  $x$  between  $0$  and  $-1$ , which can be parametrized by  $E_{\text{sym}}(\rho) \approx 31.6(\rho/\rho_0)^\gamma$  with  $\gamma = 0.69 - 1.05$  at subnormal density ( $\rho \leq \rho_0$ ) [45, 58, 64, 65], and has led to the extraction of  $61 \text{ MeV} \leq L \leq 111 \text{ MeV}$  and  $-82 \text{ MeV} \leq K_{\text{sym}} \leq 101 \text{ MeV}$  [45, 58, 64, 65]. Using the Skyrme interactions consistent with the EOS obtained from the MDI interaction with  $x$  between  $0$  and  $-1$ , the neutron-skin thickness of heavy nuclei calculated within the Hartree-Fock approach is consistent with available experimental data [45, 66] and also that from a relativistic mean-field model based on an accurately calibrated parameter set that reproduces the GMR in  $^{90}\text{Zr}$  and  $^{208}\text{Pb}$  as well as the isovector giant dipole resonance of  $^{208}\text{Pb}$  [67]. The extracted symmetry energy further agrees with the symmetry energy  $E_{\text{sym}}(\rho) = 31.6(\rho/\rho_0)^{0.69}$  recently obtained from the isoscaling analyses of isotope ratios in intermediate energy heavy ion collisions [68], which gives  $L \approx 65 \text{ MeV}$  and  $K_{\text{sym}} \approx -61 \text{ MeV}$ . Furthermore, it is interesting to mention that the above limited range of  $E_{\text{sym}}(\rho)$

at subsaturation density is essentially consistent with the symmetry energy  $E_{\text{sym}}(\rho) = 12.5(\rho/\rho_0)^{2/3} + 17.6(\rho/\rho_0)^\gamma$  with  $\gamma = 0.4 - 1.05$ , extracted very recently from analyses using the ImQMD (Improved QMD) model which can reproduce both the isospin diffusion data and the double neutron/proton ratio simultaneously [69]. The symmetry energy  $E_{\text{sym}}(\rho) = 12.5(\rho/\rho_0)^{2/3} + 17.6(\rho/\rho_0)^\gamma$  with  $\gamma = 0.4 - 1.05$  thus leads to the constraints of  $46 \text{ MeV} \leq L \leq 80 \text{ MeV}$  and  $-82 \text{ MeV} \leq K_{\text{sym}} \leq -36 \text{ MeV}$ .

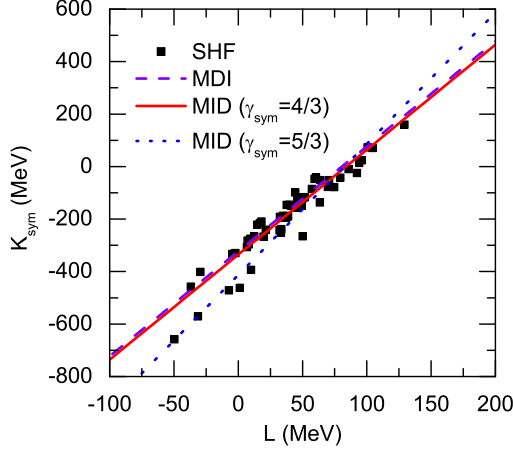


FIG. 2: (Color online) Correlation between  $K_{\text{sym}}$  and  $L$  from the MID interaction with  $\gamma_{\text{sym}} = 4/3$  and  $5/3$ , the MDI interaction and the SHF prediction with 63 popular Skyrme forces.

It should be noted that all the above constraints on  $L$  and  $K_{\text{sym}}$  are based on some unique energy density functionals and thus special correlation between  $L$  and  $K_{\text{sym}}$  has been implicitly assumed. It is thus interesting to see if there exists a universal correlation between  $L$  and  $K_{\text{sym}}$ . For the MDI interaction, the  $L$  and  $K_{\text{sym}}$  both change linearly with the parameter  $x$  and therefore they are linearly correlated by varying the parameter  $x$  [58, 70]. Similarly, for the MID interaction, one can see from Eq. (21) and Eq. (22) that the  $L$  and  $K_{\text{sym}}$  both change linearly with the parameter  $y$ , and thus they are also linearly correlated by varying the parameter  $y$ . In particular, we have

$$K_{\text{sym}} = 3\gamma_{\text{sym}}L + E_{\text{sym}}^{\text{kin}}(\rho_0)(3\gamma_{\text{sym}} - 2) - 9\gamma_{\text{sym}}E_{\text{sym}}(\rho_0). \quad (23)$$

Also the  $L$  and  $K_{\text{sym}}$  are expected to be correlated within the SHF energy density functional. Shown in Figure 2 are the correlation between  $K_{\text{sym}}$  and  $L$  from the MID interaction with  $\gamma_{\text{sym}} = 4/3$  and  $5/3$  ( $E_{\text{sym}}^{\text{kin}}(\rho_0) = 12.3 \text{ MeV}$  and  $E_{\text{sym}}(\rho_0) = 30 \text{ MeV}$ ), the MDI interaction and the SHF prediction with 63 popular Skyrme forces. The 63 Skyrme forces include the 51 forces used in Ref. [70] and 12 new forces, i.e., Z,  $E_\sigma$ , E,  $Z_\sigma$ ,  $Z_\sigma^*$ , SkSC4, SI, SII, SIII, SIV, SV, and SVI. All these Skyrme forces predict the saturation density and the symmetry energy satisfying  $0.140 \text{ fm}^{-3} < \rho_0 < 0.165 \text{ fm}^{-3}$  and  $25 \text{ MeV} < E_{\text{sym}}(\rho_0) < 37 \text{ MeV}$ , respectively.

It is interesting to see that the  $K_{\text{sym}}$  parameter indeed displays approximately a linear correlation with the  $L$  parameter for the SHF prediction with the 63 Skyrme forces and this linear correlation is nicely reproduced by the MDI interaction and the MID interaction with  $\gamma_{\text{sym}} = 4/3$ . For the MID interaction, one can see from Eq. (22) that the  $\gamma_{\text{sym}}$  parameter controls the shape (slope) of the linear correlation between  $L$  and  $K_{\text{sym}}$ . Furthermore, it is seen from Figure 2 that there are a few Skyrme forces deviate from the linear correlation obtained by the MDI interaction and the MID interaction with  $\gamma_{\text{sym}} = 4/3$ . In order to consider the uncertainty of the shape (slope) for the correlation between  $L$  and  $K_{\text{sym}}$ , we thus include the result with  $\gamma_{\text{sym}} = 5/3$  for the MID interaction. The correlation between  $K_{\text{sym}}$  and  $L$  from the SHF prediction with the 63 Skyrme forces is nicely consistent with that from the MID interaction with  $\gamma_{\text{sym}} = 4/3$  and  $5/3$ . The linear correlation between  $K_{\text{sym}}$  and  $L$  implies that one can obtain  $K_{\text{sym}}$  from  $L$ .

### B. Correlation between $J_0$ and $K_0$

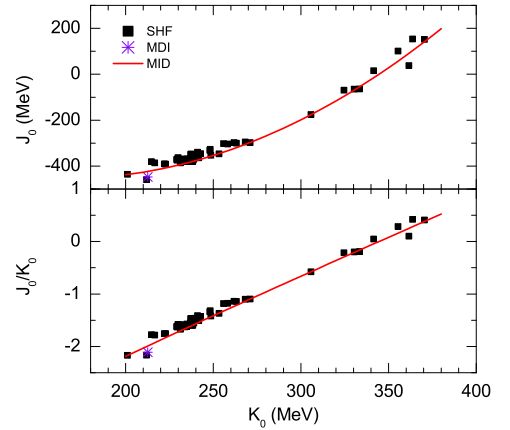


FIG. 3: (Color online)  $J_0$  and  $J_0/K_0$  as a function of  $K_0$  from the MID interaction, the MDI interaction and the SHF prediction with 63 popular Skyrme forces.

While  $K_0$  has been relatively well determined, the  $J_0$  parameter is poorly known and actually there is no any experimental information on the  $J_0$  parameter. In the MID model, from Eq. (19) one can easily calculate the  $J_0$  parameter as

$$\begin{aligned} J_0 &= 27\rho_0^3 \frac{\partial^3 E_0(\rho)}{\partial^3 \rho} \Big|_{\rho=\rho_0} \\ &= \frac{1}{70.1} (K_0^2 - 332.2K_0 - 4243.2) \text{ (MeV)}, \end{aligned} \quad (24)$$

where the unit of  $K_0$  is MeV. Therefore, in the MID model, the  $J_0$  parameter is quadratically correlated with  $K_0$ . Shown in Figure 3 are  $J_0$  and  $J_0/K_0$  as functions of  $K_0$ . Also included in Figure 3 are the corresponding results from the MDI interaction and the SHF

prediction with the 63 Skyrme forces. It is interesting to see that the correlation between  $J_0$  and  $K_0$  is quite consistent for the three different models, namely, the MID interaction, the MDI interaction and the 63 Skyrme forces in the SHF approach. In particular, the  $J_0/K_0$  displays approximately a linear correlation with  $K_0$ . This linear correlation can be easily understood from Eq. (24). On the r.h.s of Eq. (24), the last term is very small compared with the first term and the second term and thus one has  $J_0 \approx \frac{1}{70.1} (K_0^2 - 332.2K_0)$ , and then  $J_0/K_0 \approx \frac{1}{70.1} (K_0 - 332.2)$  with the unit of  $K_0$  being MeV. We note here that the correlation between  $J_0$  and  $K_0$  obtained in the present work is also consistent with the early finding by Pearson [71]. While there is no any empirical constraint on the  $J_0$  parameter, we assume in the present work the correlation between  $J_0$  and  $K_0$  from the MID interaction is valid and then we can obtain  $J_0/K_0$  from the experimental constraint on  $K_0$ .

### C. Phenomenological MID model constraint on the $K_{\text{sat},2}$ parameter

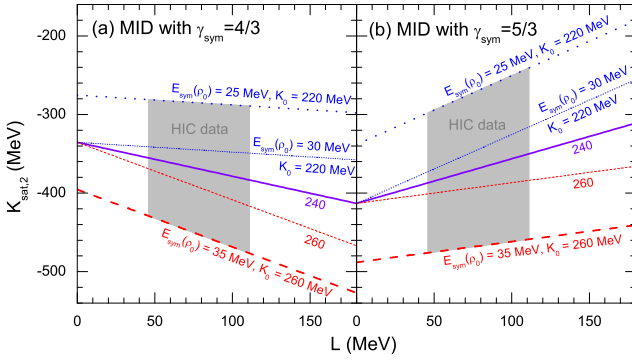


FIG. 4: (Color online)  $K_{\text{sat},2}$  as a function of  $L$  from the MID interaction with  $\gamma_{\text{sym}} = 4/3$  (a) and  $5/3$  (b) for different values of  $K_0$  and  $E_{\text{sym}}(\rho_0)$ . The shaded region indicates constraints within MID interaction with  $220 \text{ MeV} \leq K_0 \leq 260 \text{ MeV}$ ,  $25 \text{ MeV} \leq E_{\text{sym}}(\rho_0) \leq 35 \text{ MeV}$ , and  $46 \text{ MeV} \leq L \leq 111 \text{ MeV}$  limited by the heavy-ion collision data.

As shown in Eq. (9), the  $K_{\text{sat},2}$  parameter is completely determined by  $J_0/K_0$ ,  $L$ , and  $K_{\text{sym}}$ . Based on the correlations shown in Figure 2 and Figure 3, we can now extract information on the  $K_{\text{sat},2}$  parameter from the experimental constraints on the  $K_0$  parameter and the  $L$  parameter within the MID model. As pointed out previously, the value of  $K_0$  has been relatively well determined to be  $240 \pm 20 \text{ MeV}$  from the nuclear GMR [16, 17, 18, 19, 20]. The slope parameter  $L$  has been found to correlate linearly with the neutron-skin thickness of heavy nuclei and thus can in principle be determined from measured thickness of the neutron skin of such nuclei [45, 66, 72, 73, 74, 75, 76, 77]. Unfortunately, because of the large uncertainties in the experimental measurements, this has not yet been possible so far. The

proposed experiment of parity-violating electron scattering from  $^{208}\text{Pb}$ , i.e., Parity Radius Experiment (PREx) at the Jefferson Laboratory is expected to give an independent and accurate measurement of its neutron skin thickness (within 0.05 fm) [78, 79]. On the other hand, as mentioned previously, heavy-ion collisions, especially those induced by neutron-rich nuclei, provide a unique tool to explore the density dependence of the symmetry energy and thus the  $L$  parameter.

In the MID model, from Eqs. (9) and (23), we have

$$K_{\text{sat},2} = -\left(\frac{J_0}{K_0} + 6 - 3\gamma_{\text{sym}}\right)L + (3\gamma_{\text{sym}} - 2)E_{\text{sym}}^{\text{kin}}(\rho_0) - 9\gamma_{\text{sym}}E_{\text{sym}}(\rho_0). \quad (25)$$

Shown in Figure 4 is  $K_{\text{sat},2}$  as a function of  $L$  from the MID interaction with  $\gamma_{\text{sym}} = 4/3$  (panel (a)) and  $5/3$  (panel (b)) for  $K_0 = 220, 240$ , and  $260 \text{ MeV}$ . In Eq. (25),  $J_0$  can be obtained from Eq. (24) for a fixed  $K_0$  value. In the MID interaction,  $E_{\text{sym}}^{\text{kin}}(\rho_0) = 12.3 \text{ MeV}$  and  $E_{\text{sym}}(\rho_0) = 30 \text{ MeV}$  have been used as a default. From Eq. (23), one can see that the correlation of  $K_{\text{sym}}$  and  $L$  also depends on  $E_{\text{sym}}(\rho_0)$ . To consider the uncertainty due to the  $E_{\text{sym}}(\rho_0)$ , we thus also include in Figure 4 the results with  $K_0 = 220 \text{ MeV}$  and  $E_{\text{sym}}(\rho_0) = 25 \text{ MeV}$  as well as  $K_0 = 260 \text{ MeV}$  and  $E_{\text{sym}}(\rho_0) = 35 \text{ MeV}$ , which represent, respectively, the upper and lower boundaries for a fixed  $L$ . The shaded region in Figure 4 further considers the constrained  $L$  values from heavy-ion collision data, namely,  $46 \text{ MeV} \leq L \leq 111 \text{ MeV}$ . The lower limit of  $L = 46 \text{ MeV}$  is obtained from the lower boundary of the ImQMD analyses on the isospin diffusion data and the double neutron/proton ratio [69] while the upper limit of  $L = 111 \text{ MeV}$  corresponds to the upper boundary of  $L$  from the IBUU04 transport model analysis on the isospin diffusion data [45, 58, 64, 65]. The constraint  $46 \text{ MeV} \leq L \leq 111 \text{ MeV}$  is also consistent with the analyses of the pygmy dipole resonances [80], the giant dipole resonance (GDR) of  $^{208}\text{Pb}$  analyzed with Skyrme forces [81], the Thomas-Fermi model fitted very precisely to binding energies of 1654 nuclei [82], and the recent neutron-skin analysis [83]. These empirically extracted values for  $L$  represent the best and most stringent phenomenological constraints available so far on the nuclear symmetry energy at sub-saturation densities.

It is seen from Figure 4 that the  $K_{\text{sat},2}$  decreases with increasing  $L$  for  $\gamma_{\text{sym}} = 4/3$  while it increases with increasing  $L$  for  $\gamma_{\text{sym}} = 5/3$ . This feature can be easily understood from Eq. (25). For  $\gamma_{\text{sym}} = 4/3$ , Eq. (25) is reduced to

$$K_{\text{sat},2} = -\left(\frac{J_0}{K_0} + 2\right)L - 12E_{\text{sym}}(\rho_0) + 24.6 \text{ (MeV)} \quad (26)$$

while for  $\gamma_{\text{sym}} = 5/3$ , it is reduced to

$$K_{\text{sat},2} = -\left(\frac{J_0}{K_0} + 1\right)L - 15E_{\text{sym}}(\rho_0) + 36.9 \text{ (MeV)}. \quad (27)$$

For  $K_0 = 240 \pm 20$  MeV,  $J_0/K_0$  can be found from Figure 3 (or Eq. (24)) to be from about  $-1.9$  to  $-1.3$ . Therefore,  $K_{\text{sat},2}$  decreases (increases) with increasing  $L$  for  $\gamma_{\text{sym}} = 4/3$  ( $5/3$ ) following Eq. (26) (Eq. (27)).

An interesting feature observed from Figure 4 is that the  $K_{\text{sat},2}$  parameter significantly depends on the symmetry energy at the normal nuclear density  $E_{\text{sym}}(\rho_0)$ . This can be seen more clearly from Eqs. (26) and (27) which indicate that changing  $E_{\text{sym}}(\rho_0)$  by 5 MeV leads to a variation of 60 – 75 MeV for  $K_{\text{sat},2}$ . This feature indicates that an accurate determination of  $E_{\text{sym}}(\rho_0)$  is important for determining the value of  $K_{\text{sat},2}$ . From the shaded region indicated in Figure 4, it is found that for  $\gamma_{\text{sym}} = 4/3$ , we have  $-429 \text{ MeV} \leq K_{\text{sat},2} \leq -281 \text{ MeV}$  for  $L = 46 \text{ MeV}$  while  $-477 \text{ MeV} \leq K_{\text{sat},2} \leq -289 \text{ MeV}$  for  $L = 111 \text{ MeV}$ . For  $\gamma_{\text{sym}} = 5/3$ , we have  $-476 \text{ MeV} \leq K_{\text{sat},2} \leq -298 \text{ MeV}$  for  $L = 46 \text{ MeV}$  while  $-459 \text{ MeV} \leq K_{\text{sat},2} \leq -241 \text{ MeV}$  for  $L = 111 \text{ MeV}$ . These results indicate that within the MID model with the empirical constraints of  $K_0 = 240 \pm 20 \text{ MeV}$ ,  $25 \text{ MeV} \leq E_{\text{sym}}(\rho_0) \leq 35 \text{ MeV}$ , and  $46 \text{ MeV} \leq L \leq 111 \text{ MeV}$ , the  $K_{\text{sat},2}$  parameter can be varied from  $-477 \text{ MeV}$  to  $-241 \text{ MeV}$ .

As shown in Eq. (11), the  $K_{\text{asy}}$  parameter corresponds to the  $K_{\text{sat},2}$  parameter when  $J_0$  is zero, i.e., the parabolic approximation to the EOS of symmetric nuclear matter Eq. (5) is valid. From the MID model, a vanishing  $J_0$  corresponds to a  $K_0$  value of about 340 MeV, which is significantly larger than the empirical value of  $240 \pm 20 \text{ MeV}$ . In the MID model, we have  $J_0/K_0 \approx -1.6$  for  $K_0 = 240 \text{ MeV}$  and thus  $K_{\text{sat},2} \approx K_{\text{asy}} + 1.6L$  or  $K_{\text{asy}} \approx K_{\text{sat},2} - 1.6L$ . Therefore, the difference between  $K_{\text{asy}}$  and  $K_{\text{sat},2}$  depends on  $L$  with a larger  $L$  value (stiffer symmetry energy) leading to larger difference. At this point, it should be stressed that the  $K_{\text{asy}}$  parameter is completely determined by the density dependence of the symmetry energy regardless of the EOS of symmetric nuclear matter. Based on the IBUU04 transport model analysis on the isospin diffusion data [58, 65], a value of  $K_{\text{asy}} = -500 \pm 50 \text{ MeV}$  has been extracted from the symmetry energy obtained by the MDI interaction with the  $x$  parameter between 0 and  $-1$ . The constraint  $K_{\text{asy}} = -500 \pm 50 \text{ MeV}$  is quite consistent with the very recent constraint of  $K_{\text{asy}} \approx -500^{+125}_{-100} \text{ MeV}$  from the study of neutron skin of finite nuclei [83]. Furthermore, in the MDI interaction, we have  $-311 \text{ MeV} \leq K_{\text{sat},2} \leq -316 \text{ MeV}$  from the prediction of the MDI interaction with the  $x$  parameter between 0 and  $-1$ . Therefore, for the MDI interaction, the magnitude of  $K_{\text{sat},2}$  is significantly smaller than that of  $K_{\text{asy}}$  and is quite insensitive to the density dependence of the symmetry energy. These features indicate that the high-order  $J_0$  contribution to  $K_{\text{sat},2}$  generally cannot be neglected.

#### IV. SUMMARY AND CONCLUSIONS

We have constructed a phenomenological momentum-independent MID model which can reasonably describe

the general properties of symmetric nuclear matter and the symmetry energy predicted by both the sophisticated isospin and momentum dependent MDI model and the SHF approach with different Skyrme forces. In particular, the density functional of the symmetry energy constructed in the MID model is shown to be very flexible and can mimic very different density behaviors by varying only one parameter.

Based on the MID model, we have studied in detail the second-order isospin coefficient  $K_{\text{sat},2}$  which is determined uniquely by  $L$ ,  $K_{\text{sym}}$  and  $J_0/K_0$ . Our results indicate that the high-order  $J_0$  contribution to  $K_{\text{sat},2}$  generally cannot be neglected, especially for larger  $L$  values. In addition, interestingly, it is found that there exists a nicely linear correlation between  $K_{\text{sym}}$  and  $L$  as well as between  $J_0/K_0$  and  $K_0$  for the three different models used here, i.e., the MDI interaction, the MID interaction, and the SHF approach with 63 Skyrme forces. From the MID model, the correlation between  $K_{\text{sym}}$  and  $L$  is further shown to depend significantly on the value of  $E_{\text{sym}}(\rho_0)$ . These correlations and features enable us to extract the values of the  $J_0$  parameter and the  $K_{\text{sym}}$  parameter from the empirical information on  $K_0$ ,  $L$  and  $E_{\text{sym}}(\rho_0)$ . In particular, using the empirical constraints of  $K_0 = 240 \pm 20 \text{ MeV}$ ,  $25 \text{ MeV} \leq E_{\text{sym}}(\rho_0) \leq 35 \text{ MeV}$ , and  $46 \text{ MeV} \leq L \leq 111 \text{ MeV}$  in the MID model leads to an estimate of  $-477 \text{ MeV} \leq K_{\text{sat},2} \leq -241 \text{ MeV}$ .

While the estimated value of  $-477 \text{ MeV} \leq K_{\text{sat},2} \leq -241 \text{ MeV}$  in the present work has a small overlap with the constraint of  $K_\tau = -550 \pm 100 \text{ MeV}$  obtained in Ref. [18, 19] from recent measurements of the isotopic dependence of the GMR in even-A Sn isotopes, the magnitude of the constrained  $K_\tau$  is still significantly larger than that of  $-477 \text{ MeV} \leq K_{\text{sat},2} \leq -241 \text{ MeV}$ . Recently, there are several works [49, 50] on extracting the value of the  $K_{\text{sat},2}$  parameter based on the idea initiated by Blaizot and collaborators that the values of both  $K_0$  and  $K_{\text{sat},2}$  should be extracted from the same consistent theoretical model that successfully reproduces the experimental GMR energies of a variety of nuclei. These studies show that there is no a single model (interaction) which can simultaneously describe correctly the recent measurements of the isotopic dependence of the GMR in even-A Sn isotopes and the GMR data of nuclei  $^{90}\text{Zr}$  and  $^{208}\text{Pb}$ , which makes it difficult to accurately determine the value of  $K_{\text{sat},2}$  from the experimental GMR energies of a variety of finite nuclei. As pointed out in [50], these features seem to suggest that the  $K_\tau = -550 \pm 100 \text{ MeV}$  obtained in Ref. [18, 19] may suffer from the same ambiguities already encountered in earlier attempts [61] to extract the  $K_0$  and  $K_{\text{sat},2}$  of infinite matter from finite-nuclei extrapolations. This problem remains as an open challenge, and both experimental and theoretical insights are needed in the future.

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